

SOLID AND OCEAN EARTH TIDES AND THE DETECTION OF SOME GRAVITOMAGNETIC EFFECTS

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The detection of some tiny gravitomagnetic effects in the field of the Earth by means of artificial satellites is a very demanding task because of the various other perturbing forces of gravitational and non-gravitational origin acting upon them. Among the gravitational perturbations a relevant role is played by the Earth solid and ocean tides. In this communication I outline their effects on the detection of the Lense-Thirring drag of the orbits of LAGEOS and LAGEOS II, currently analyzed, and the proposed GP-C experiment devoted to the measurement of the clock effect.

1 The Lense-Thirring drag of the orbits of the LAGEOSs

The Lense-Thirring effect^{1,2} is currently measured³ by analyzing the following combination of the orbital residuals of the nodes Ω of LAGEOS and LAGEOS II and the perigee ω of LAGEOS II⁴:

$$\delta\dot{\Omega}^I + c_1\delta\dot{\Omega}^{II} + c_2\delta\dot{\omega}^{II} \simeq 60.2\mu_{LT}, \quad (1)$$

in which $c_1 = 0.295$, $c_2 = -0.35$ and μ_{LT} is the parameter to be measured which is 1 in general relativity and 0 in Newtonian mechanics. The general relativity forecasts for the combined residuals a linear trend with slope of 60.2 mas/y.

1.1 The solid tides

Concerning the solid tides, the most powerful constituents turn out to be the semisecular 18.6-year, the K_1 and the S_2 which induce on the considered LAGEOSs' elements perturbations of the order of $10^2 - 10^3$ mas with periods ranging from 111.24 days for the S_2 on LAGEOS II to 6798.38 days for the 18.6-year tide.

The latter, with its amplitudes of -1079.38 mas, 1982.16 mas and -1375.58 mas for the nodes of the LAGEOSs and the perigee of LAGEOS II respectively, could be particularly insidious for the detection of the Lense-Thirring effect. Indeed, since the observational periods adopted until now³ range only from 3.1 to 4 years, it could resemble to a superimposed linear trend which may alias

the recovery of μ_{LT} . However, its effect should vanish since it is a $l = 2$ $m = 0$ tide, and eq.(1) should be not sensible to such tides⁴. This feature will be quantitatively assessed later.

Also the K_1 plays a not negligible role: it induces on LAGEOS' s node a perturbation with period of 1043.67 days and amplitude of 1744.38 mas, while the node and the perigee of LAGEOS II are shifted by an amount of -398 mas and 1982.14 mas, respectively, over a period of -569.21 days.

1.2 The ocean tides

About the ocean tides, whose knowledge is less accurate than that of the solid tides, the $l = 3$ part of the tidal spectrum turns out to be very interesting for the perigee of LAGEOS II.

Indeed, for this element the K_1 $l = 3$ $p = 1, 2$ $q = -1, 1$ terms induce perturbations which are of the same order of magnitude of those generated by the solid tides, both in the amplitudes and in the periods. E.g., the $l = 3$ $p = 1$ $q = -1$ harmonic has a period of 1851.9 days (5.09 years) and an amplitude of 1136 mas, while the $l = 3$ $p = 2$ $q = 1$ harmonic is less powerful with its 346.6 mas and a period of -336.28 days.

It should be considered that, contrary to the first two even degree zonal perturbations which do not affect eq.(1), the diurnal odd degree tidal perturbations are not canceled out by the combined residuals of Ciufolini. So, over an observational period of few years the K_1 $l = 3$ $p = 1$ $q = -1$ harmonic may alias the gravitomagnetic trend of interest.

The even degree ocean tidal perturbations are not particularly relevant: they amount to some tens of mas or less, with the exception of K_1 which perturbs the node of LAGEOS and the perigee of LAGEOS II at a level of 10^2 mas.

2 The effect of the orbital tidal perturbations on the measurement of the Lense-Thirring effect

Given an observational period T_{obs} , the tidal perturbations must be divided into two main categories according to their periods P : those with $P < T_{obs}$ and those with $P > T_{obs}$. While the former ones, even if their mismodeled amplitude is great so that they heavily affect the orbital residuals, are not particularly insidious because their effect averages out if $T_{obs} = nP$, $n = 1, 2, \dots$, the latter ones, on the contrary, are particularly hazardous since they may alter the determination of μ_{LT} acting as superimposed bias. This is particularly true for those diurnal and semidiurnal tides which should affect the combined residuals, as the K_1 $l = 3$ $m = 1$ $p = 1$ $q = -1$.

A preliminary analysis has been conducted by calculating eq.(1) with the nominal tidal perturbative amplitudes worked out in the previous sections for $T_{obs} = 1$ year. The calculations have been repeated also with the mismodeled amplitudes. They show that, not only the $l = 2, 4$ $m = 0$ tides tend to cancel out, but also that this feature extends to the $l = 3$ $m = 0$ ocean tides.

A more refined procedure will be described below for the diurnal and semidiurnal tides.

2.1 Case a: $P < T_{obs}$

The effect of such class of perturbations has been evaluated as follows.

The orbital residuals curve has been simulated with MATLAB by including the Lense-Thirring trend as predicted by general relativity, the main mismodeled tidal perturbations in the form of:

$$\delta A_f \sin\left(\frac{2\pi}{P_f}t + \phi_f\right), \quad (2)$$

where f denotes the harmonic chosen, and a noise. About the mismodeling δA , we have assumed that the main source of uncertainties are the free space potential Love number k_2 ⁵ for the solid tides, the load Love numbers⁶ and the ocean tidal heights C_{lmf}^+ ⁷ for the ocean tides. The MATLAB routine has been endowed with the possibility of changing the time series length T_{obs} , the time sampling Δt , the amplitude of the noise, and the harmonics' initial phases ϕ .

The so obtained simulated curves, for different choices of T_{obs} , have been subsequently fitted with a least-square routine in order to recover, among the other things, the parameter μ_{LT} . This procedure has been repeated with and without the whole set of mismodeled tidal signals so to obtain an estimate $\Delta\mu_{tides} = \mu_{LT}(all\ tides) - \mu_{LT}(no\ tides)$ of their influence on the measurement of μ_{LT} . This analysis show that $2\% < \Delta\mu_{tides} < 4\%$ for T_{obs} ranging from 4 years to 7 years.

2.2 Case b: $P > T_{obs}$

The effect of such class of perturbations has been evaluated with different approaches.

Firstly, in a very conservative way, it has been considered the averaged value of the mismodeled tidal signal under consideration over different T_{obs} . The analysis has been performed for the 18.6-year tide and the K_1 $l = 3$ $p = 1$ $q = -1$. On the semisecular tide it has been assumed a mismodeling level of 1.5% due to the uncertainty at its frequency of the anelastic behavior of the

Earth's mantle accounted for by the Love number k_2 . The oceanic constituent has been considered unknown at a level of almost 6% due to the uncertainties on the load Love number of degree $l = 3$ and to the tidal height coefficient C_{lmf}^+ as released by EGM96. Over different T_{obs} the latter affects the determination of the Lense-Thirring effect at a level of 2.3% at most, while the zonal tide, as predicted by Ciufolini⁴, almost cancels out giving rise to negligible contributions to the combined residuals.

3 The clock effect

In the gravitomagnetic clock effect^{8 9 10 11} two clocks moving along pro- and retrograde circular equatorial orbits, respectively, about the Earth exhibit a difference in their proper times which, if calculated after some fixed angular interval, say 2π , amounts to almost 10^{-7} s

In^{9 12} it has been shown that for an orbit radius of 7000 km the radial and azimuthal locations of the satellites must be known at a level of accuracy of $\delta r \leq 10^{-1}$ mm and $\delta\phi \leq 10^{-2}$ mas per revolution.

3.1 The systematic radial errors induced by the tidal perturbations

About the radial direction, the even degree part of the tidal spectrum does not affect it contrary to the odd degree part¹³. For the $l = 3$ $m = 1$ $p = 1$ $q = -1$ ocean tides we have:

$$\Delta r_{311-1f} = (8.80 \cdot 10^{25} \text{ cm}^{7/2} \text{ s}^{-1}) \times a^{-7/2} \times P_{pert} \times C_{31f}^+, \quad (3)$$

where a is the satellite's semimajor axis and P_{pert} is the perturbation's period.

For K_1 , the most powerful diurnal ocean tide, we have $P_{pert} = 50$ days and $\delta r_{311-1}(K_1) \simeq 2.07$ cm by assuming an uncertainty of 5.2 % on the tidal height coefficient⁷. Despite the amplitude of this long period mismodeled perturbation is 2 orders of magnitude greater than the maximum allowable error $\delta r_{max} = 10^{-1}$ mm, it must be noted that its period P_{pert} amounts to only 50 days. This implies that if an observational time span T_{obs} which is an integer multiple of P_{pert} , i. e. some months, is adopted the tidal perturbative action of K_1 can be averaged out.

3.2 The systematic azimuthal errors induced by the tidal perturbations

The azimuthal angle is perturbed only by the even part of the tidal spectrum¹³. For $l = 2$ we have:

$$\Delta\phi^{solid} = (-3.77 \cdot 10^{18} \text{ cm}^{5/2} \text{ s}^{-1}) \times a^{-7/2} \times k_{20}^{(0)} \times P_{pert} \times H_2^0, \quad (4)$$

$$\Delta\phi^{ocean} = (-4.707 \cdot 10^{17} \text{ cm}^{5/2} \text{ s}^{-1}) \times a^{-7/2} \times P_{pert} \times C_{lmf}^+, \quad (5)$$

where H_l^m are the Doodson coefficients with a different normalization^{14 15}. For $a = 7000 \text{ km}$ we have:

- $\Delta\phi(18.6 - \text{year}) = -4.431 \cdot 10^4 \text{ mas}$
- $\Delta\phi(9.3 - \text{year}) = -214.4 \text{ mas}$
- $\Delta\phi(S_a) = 408 \text{ mas (solid); } 857.6 \text{ mas (oceanic)}$

The zonal tidal perturbations on the satellite's azimuthal location are particularly insidious not only because their nominal amplitudes are up to 6 orders of magnitude greater than the maximum allowable error $\delta\phi_{max} = 10^{-2} \text{ mas}$, but also because they have periods very long, so that there is no hope they average out on reasonable T_{obs} . Concerning the 18.6-year tide, by assuming an uncertainty of 1.5% on $k_{20}^{(0)}$, the mismodeling on its perturbation amounts to -664 mas which is, however, very far from $\delta\phi_{max}$.

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